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Modeling bitcoin price volatility using the ARCH model

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***Abstract*---**This study aims to identify volatility models for the daily closing price of Bitcoin in USD during the period from January 1, 2020, to June 30, 2024, by applying autoregressive conditional heteroskedasticity (ARCH) models, where the error distribution follows the normal distribution. These models take into account price fluctuations during the trading period. The results indicate that the best model for estimating the time series data of the daily closing price of Bitcoin is the EGARCH model, among other ARCH models, as it has the lowest value for the statistical criteria used (H-Q, SIC, and AIC) for model selection. This confirms the importance of using ARCH models in volatility (risk) analysis, leading to accurate and reliable conclusions that benefit market participants. Additionally, the results show the presence of variance effects on the time series of daily closing prices, which was confirmed by the ARCH test on residuals. This implies that there are fluctuations in the daily closing prices of Bitcoin, necessitating the use of ARCH family models to predict daily Bitcoin closing prices.

***Keywords*---**Bitcoin, Volatility modelling, ARCH model.

Introduction

Bitcoin is considered one of the most important virtual cryptocurrencies, available only in digital form with no physical existence. It is used through computers and electronic wallets, and is a decentralized currency used for purchasing goods and services. Unlike traditional currencies, Bitcoin is not governed by central banks but is subject to supply and demand. Bitcoin was designed with a limited supply, contrary to traditional currencies, which can be increased or decreased by

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monetary authorities. The supply of Bitcoin was set by Satoshi Nakamoto at a maximum of twenty-one million (21 million) units. Due to its decentralized nature and lack of official backing, it has been banned in many countries. In Algeria, the law prohibits its purchase, sale, usage, and possession, with strict penalties for violations. From this, the study emerged to answer the following questions:

1) How effective are ARCH models in predicting Bitcoin price volatility?
2) Can Bitcoin price volatility be modeled using Autoregressive Conditional Heteroskedasticity (ARCH) models?

Importance of the Study

The importance of this study lies in the increasing value of Bitcoin and the rising demand for it due to the advantages it offers to its users. Understanding the factors influencing Bitcoin price volatility is crucial, especially considering the concerns surrounding it, as many countries have banned it and penalized its users according to their national laws.

Objectives of the Study

The aim of this study is to shed light on Bitcoin in terms of its concept, origin, features, issuance, and usage. Furthermore, it seeks to model Bitcoin price volatility from 2020 to June 2024 using ARCH models and to determine the most suitable model for diagnosing Bitcoin price fluctuations, identifying the reasons behind these fluctuations, the most influential factors, and the resulting implications.

Methodology and Data

This study employs a descriptive-analytical method for the theoretical aspect and an econometric approach for the applied analysis. The data used are daily Bitcoin prices in U.S. dollars, obtained from the website https://finance.yahoo.com. The dataset includes daily closing prices from January 1, 2020, to June 30, 2024, with a total of 1,640 observations.

Structure of the Study

To achieve the research objectives and address the problem, the study is structured as follows: the first section covers the econometric model and steps for building it, while the second section focuses on the applied analysis. The conclusion will summarize the findings and provide recommendations.

Section One: The Econometric Model and Steps for Model Building

First: Building the Econometric Model
Time series of trading indicators often experience significant volatility due to political and economic factors, which, in turn, influence market investments. Various criteria are used to diagnose the most suitable model:

1. **Akaike Information Criterion (AIC):**
   Proposed by Hirotugu Akaike in 1974, the AIC criterion, denoted as AIC, is used to select the appropriate model rank from among several models. The

rank corresponding to the lowest AIC value is the most appropriate for the observations. The AIC formula can be expressed as follows (Akaike, 1974):
AIC= (-2) Ing (maximum likelihood) +2k
Where: K represents the number of model parameters.

2. **Schwarz Information Criterion (SIC):**
In 1978, Gideon Schwarz introduced the Schwarz Information Criterion (SIC), represented by SIC, as a refinement to address the inconsistency found in AIC. Schwarz imposes a stricter penalty, expressed as kln⁡(n)k \ln(n)kln(n), and the SIC formula is given as follows (Hassan, 2017):
SIC = -2(Maximum likelihood)+kIn(n)
The model rank corresponding to the lowest SIC value is selected.
3. **Hannan-Quinn Criterion (H-Q):**
In 1979, Hannan and Quinn proposed a new criterion for model selection, known as the Hannan-Quinn criterion (H-Q), denoted by H−QH-QH−Q. The mathematical expression is as follows (Manal Belkacem, 2021):
C>2H $H - Q = ln\hat{\sigma}_a^2 + 2hCln(\text{In}\, n)/n$
The model rank corresponding to the lowest H-Q value is selected.

Second: Estimating ARCH and GARCH Models

ARCH and GARCH models are used in financial data analysis to model variance. Modern investors are not only concerned with predicting expected returns on bonds and stocks but are also interested in risk factors and uncertainty. To study uncertainty, we must address volatility in stock prices over time, known as variance. The models that handle this type of variance belong to the ARCH family (Hussein Batal, 2020).

Most commonly used volatility prediction models belong to the GARCH family. The first model of this type, Autoregressive Conditional Heteroskedasticity (ARCH), was proposed by Engle in 1982. The Generalized ARCH (GARCH) model by Bollerslev (1986) became the foundation for most volatility models. Since then, a rich family of GARCH models has emerged, though their usage is often limited.
GARCH models fall under the category of conditional volatility models, which rely on the optimal exponential weighting of historical returns to assign less weight to more recent returns. The model parameters are typically estimated using maximum likelihood (Jón Daníelsson, 2011).

01 - The ARCH Model:

The first model that provides a methodological framework for modeling volatility is the ARCH model by Engle (1982). The fundamental idea behind ARCH models is that (a) the shock ata_tat resulting from asset returns is not serially correlated but is dependent. And (b) the dependence of ata_tat can be described through a simple quadratic function of its lagged values. Specifically, the ARCH(m) model assumes that:

$$a_t = \sigma_t \epsilon_t \qquad a_t^2 = \alpha_0 + \alpha_1 a_t^2 + \cdots + \alpha_m a_t^2$$

Where $\{\epsilon_t\}$ represents a series of independent and identically distributed (iid) random variables with a mean of zero and a variance of one, with $1\alpha_0 > 0$, $\mathrm{i} >$

0 ـل $\alpha_i \geq 0$ و. The parameters ai must satisfy certain regularity conditions to ensure that the unconditional variance of a_t is finite. Statistically, ϵ_t is often assumed to follow a standard normal distribution, a standardized Student-t distribution, or a generalized error distribution.

From the model's structure, it is evident that large past squared shocks $\{a_{t-i}^2\}_{i=1}^{m}$ imply a large conditional variance of a_t^2 for the innovation a_t and thus a_t tends to take on a large value (in magnitude). This means that, within the ARCH framework, large shocks tend to be followed by another large shock. The word "tend" is used here because large variance does not necessarily lead to large shocks; it merely indicates a higher likelihood of experiencing a large variance compared to a smaller one. This feature resembles the volatility clusters observed in asset returns (Ruey S. Tsay, 2005).

02 - The GARCH Model:

One of the weaknesses of the ARCH model is that it often requires many parameters and a high order to capture the volatility process. To address this shortcoming, Bollerslev (1986) proposed the GARCH model, which is based on an infinite ARCH specification. This allows for the reduction of the number of estimated parameters by imposing nonlinear constraints. The standard GARCH(p, q) model expresses the variance at time t، σ_t^2 as follows: (Alberga, Shalita, Yosef 2008).

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \sigma_i \varepsilon_{t-i}^2 + \sum_{j-1}^{p} \beta_j \sigma_{t-j}^2$$

Where β_j، σ_i ωو are the parameters to be estimated, and by using the lag operator L, the variance becomes:

$$\sigma_t^2 = \omega + \alpha(L)\varepsilon_t^2 + \beta(L)\sigma_t^2$$

Where : $\alpha(L) = \sum_{i=1}^{q} \alpha_i L^i$ و $\beta(L) = \sum_{j=1}^{p} \beta_j L^j$.

If all the polynomial roots $|1 - \beta(L)| = 0$ lie outside the unit circle, then we have:

$$\sigma_t^2 = \omega[1 - \beta(L)]^{-1} + \alpha(L)[1 - \beta(L)]^{-1}\varepsilon_t^2$$

This equation can be perceived as an ARCH(∞) process since the conditional variance linearly depends on all previous squared residual values. Consequently, the conditional variance of y_t can become greater than the unconditional variance. Therefore, if the previous achievements of ε_t^2 from σ^2, the variance will be given by:

$$\sigma^2 = E(\varepsilon_i^2) = \frac{\omega}{1 - \sum_{i=1}^{q} \alpha_i - \sum_{j=1}^{p} \beta_j}$$

Similar to ARCH, some constraints are necessary to ensure that σ_t^2 is positive for all t. Bollerslev demonstrates that imposing ω>0، ai ≥0 ensures this condition.
(For i=1 , …….,q0 و (βj≥) j=1,……,p). It is sufficient for the conditional variance to be positive.

03 - The EGARCH Model:

This is another commonly used asymmetric GARCH model developed by Nelson in 1991. The exponential GARCH (EGARCH) model can be defined as follows (Wang P., 2009):

$$\text{In}(\sigma_t^2) = \alpha_0 + \beta \text{In}\,(\sigma_{t-1}^2) + \alpha \left\{ \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| - \sqrt{\frac{2}{\pi}} \right\} - \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}}$$

Where γ is the asymmetric response parameter or leverage parameter. It is expected that the sign of γ will be positive in most empirical cases, meaning that a negative shock increases future volatility or uncertainty, while a positive shock affects future uncertainty. This contrasts with the standard GARCH model, where shocks of the same magnitude, whether positive or negative, have the same effect on future volatility in macroeconomic analysis, financial markets, and corporate finance.

04 - The TGARCH Model:

The TGARCH model was studied by Zakoian in 1994. The specification of the TGARCH model is based on the conditional standard deviation rather than the conditional variance (P. Sidorov, 2010).

$$\sigma_t^2 => \alpha_0 + \alpha_1^+ |\varepsilon_{t-1}^+|^2 - \alpha_1^- |\varepsilon_{t-1}^-|^2 + \delta\sigma_{t-1}^2$$

The phenomenon of shock asymmetry is tested through the following null hypothesis:
$H_0 \Rightarrow \alpha_1^- = 0$ (symmetry of the effects of negative and positive shocks on volatility, indicating no difference).

Second Section: Modeling Bitcoin Price Volatility Using ARCH Models

In this section, we will calculate Bitcoin prices using ARCH models. To achieve this, we will rely on the following stages: a descriptive study of the return series of Bitcoin prices, followed by a study of the stability of Bitcoin prices, then conducting ARCH tests to verify the validity of the model, and finally selecting the best ARCH model to determine the volatility series of Bitcoin prices, followed by an assessment of the model's adequacy.

First - Descriptive Study of the Bitcoin Price Return Series:

In the applied aspect, we relied on daily closing price data for Bitcoin to extract its volatility over the period from January 1, 2020, to June 30, 2024, with a total of 1,643 observations. In the following figure,

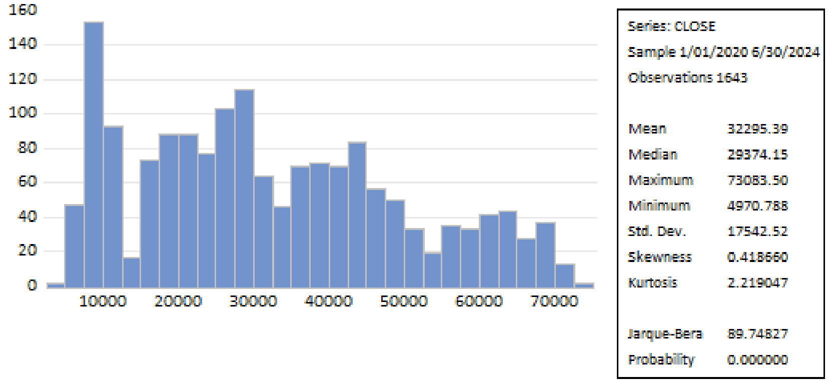

Figure 01: Descriptive Statistics for the Data Series (Jarque-Bera Test) for Normal Distribution

Source: Prepared by the researchers using EViews 13.

According to Figure 01, it is evident that the average of the Bitcoin series is 32,295.39, with the maximum value reaching 73,083 and the minimum value being 4,970.788. The standard deviation is 17,542.52, and the skewness coefficient is (Skewness=0.418660), indicating a positive skewness, meaning that the error distribution has a long tail to the right. Additionally, we note that the kurtosis value is (kurtosis=2.219047), which differs from the value of 333, indicating that the distribution is biased towards a value lower than that of the normal distribution, suggesting that the residuals have moderately flat tails. As for the Jarque-Bera value of 89.74827, it is significant at the 1% level, which indicates that, based on all these indicators, we can infer that the residuals do not follow a normal distribution.

Second - Stability Study of the Series:

To study any economic phenomenon, it is essential to ensure the stability of the series representing that phenomenon. Afterward, we can estimate the model that represents the phenomenon, as the time series of daily prices may be unstable. To test for the stability of the series, we must use the Augmented Dickey-Fuller test and the Phillips-Perron test.

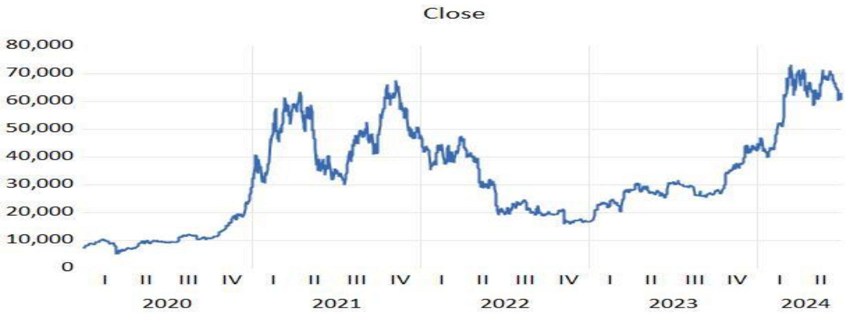

Figure 02: Curve of the Bitcoin Series

Prepared by the researchers using the statistical software EViews-13.
It is clear from the series curve that it is unstable and characterized by significant fluctuations. From the beginning of 2020 to 2021, the currency experienced a continuous increase, followed by fluctuations during the period from 2021 to 2022, and then a noticeable decline, reaching its lowest point in mid-2022. Subsequently, the currency began to improve and increase until 2024. Thus, we conclude that the curve has a trend component and is unstable at its original level.

Date: 07/23/24 Time: 13:53
Sample: 1/01/2020 6/30/2024
Included observations: 1643

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.996	0.996	1632.9	0.000
		2	0.992	0.048	3255.0	0.000
		3	0.989	-0.003	4866.4	0.000
		4	0.985	-0.029	6466.2	0.000
		5	0.981	-0.023	8054.1	0.000
		6	0.977	-0.012	9629.9	0.000
		7	0.973	0.009	11194.	0.000
		8	0.969	-0.013	12746.	0.000
		9	0.965	0.023	14286.	0.000
		10	0.961	-0.025	15815.	0.000
		11	0.957	-0.020	17331.	0.000
		12	0.952	-0.021	18834.	0.000
		13	0.948	0.026	20326.	0.000
		14	0.944	-0.024	21804.	0.000
		15	0.940	-0.031	23270.	0.000
		16	0.935	0.006	24723.	0.000
		17	0.931	0.017	26163.	0.000
		18	0.926	-0.017	27591.	0.000
		19	0.922	-0.015	29005.	0.000
		20	0.918	0.021	30407.	0.000
		21	0.913	-0.006	31797.	0.000
		22	0.909	-0.011	33173.	0.000
		23	0.904	-0.007	34538.	0.000
		24	0.900	0.023	35890.	0.000
		25	0.895	-0.040	37229.	0.000
		26	0.891	-0.026	38555.	0.000
		27	0.886	-0.014	39867.	0.000
		28	0.881	-0.017	41165.	0.000
		29	0.876	-0.038	42449.	0.000
		30	0.870	0.013	43719.	0.000
		31	0.865	0.002	44975.	0.000
		32	0.860	-0.033	46215.	0.000
		33	0.855	0.029	47443.	0.000
		34	0.850	-0.046	48655.	0.000
		35	0.844	-0.007	49853.	0.000
		36	0.839	0.013	51036.	0.000

Figure 03: Autocorrelation Function of the Bitcoin Series

Prepared by the researchers using the statistical software EViews 13.
From Figure 03, it is evident that all Q-Stat values are significant and differ from zero, indicating that the autocorrelation parameters are very high, approaching

0.99, and have exceeded the confidence limits. This suggests that our series is unstable.

To verify the preliminary results regarding the absence of stability in the original form of the study variables according to graphical analysis and autocorrelation studies, the next step during this period will involve subjecting the time series to the Augmented Dickey-Fuller (ADF) unit root test. The following table summarizes the results of this test.

Table 01: Unit Root Test Results for Phillips-Perron and Dickey-Fuller

Tests	Level	First Difference
	Constant Only	Constant and Trend
ADF	-1.2505 Prob=0.6543	-1.5204 Prob=0.8225
PP	-1.2443 Prob=0.6571	-1.5157 Prob=0.8242

Source: Prepared by the researchers using EViews 13.

From the above table, we observe that the daily closing prices of Bitcoin are unstable at the level for both models (ADF + PP), as the p-values for the t-stat test are less than 0.05. However, the series stabilizes when the first difference is applied. The first difference was selected due to the instability of the series at the level, and the series is designated as (dcl).

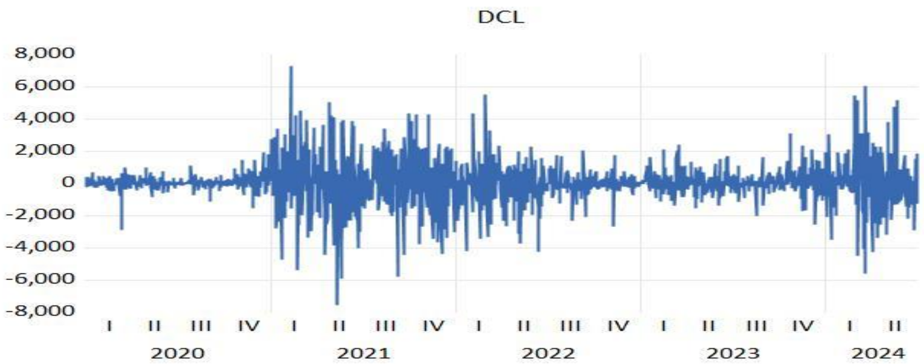

Figure 03: Graph of the dcl Series
Source: Prepared by the researchers using EViews 13.

We observe from Figure (3) that the trend component has been removed from the series, resulting in stability, as the mean appears to be constant over time. It is noted that the series goes through periods of low volatility followed by periods of high volatility, which is characteristic of financial time series.

Third: Testing ARCH Models:

ARCH models allow for the modeling of series that exhibit rapid instantaneous fluctuations dependent on the past. The presence of an ARCH effect relies on the possibility of a self-regression among the squared daily prices. This can be demonstrated either by examining the autocorrelation and partial autocorrelation or by conducting a Lagrange multiplier test or performing a Ljung-Box test, which are used to assess the randomness of the series' residuals by calculating the autocorrelation coefficients of the residuals.

01- Testing for the Presence of an ARCH Effect:

Table 02: ARCH Test for Bitcoin Price Series

Heteroskedastisticity Test ARCH			
F-statistic	30.2623	Prob.F(1,1638)	0.000
Obs*R-squared	29.7496	Prob.Chi-square(1)	0.000

Source: Prepared by the researchers using Eviews 13

From the table, we observe that for the Fisher F-statistic and the Lagrange multiplier, the probability values are negligible, being less than 5%. This indicates that we reject the null hypothesis of the absence of an ARCH effect and accept the existence of an ARCH effect, meaning that there is an impact of heteroskedasticity.

02 - Estimation of the GARCH Model

Dependent Variable: DCL
Method: ML ARCH - Normal distribution (Marquardt / EViews legacy)
Date: 08/21/24 Time: 00:38
Sample (adjusted): 1/03/2020 6/30/2024
Included observations: 1641 after adjustments
Convergence achieved after 52 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	39.05391	16.68935	2.340049	0.0193
DCL(-1)	-0.030969	0.025622	-1.208678	0.2268
Variance Equation				
C	4126.487	457.5275	9.019101	0.0000
RESID(-1)^2	0.083308	0.006579	12.66353	0.0000
GARCH(-1)	0.921438	0.005377	171.3559	0.0000

R-squared	0.002872	Mean dependent var	33.93834
Adjusted R-squared	0.002264	S.D. dependent var	1228.217
S.E. of regression	1226.826	Akaike info criterion	16.41139
Sum squared resid	2.47E+09	Schwarz criterion	16.42785
Log likelihood	-13460.54	Hannan-Quinn criter.	16.41749
Durbin-Watson stat	2.060095		

Source: Prepared by the researchers using Eviews 13

03 - Estimation of the TGARCH Model

Dependent Variable: DCL
Method: ML ARCH - Normal distribution (Marquardt / EViews legacy)
Date: 08/21/24 Time: 00:52
Sample (adjusted): 1/03/2020 6/30/2024
Included observations: 1641 after adjustments
Convergence achieved after 95 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-1)^2*(RESID(-1)<0)
+ C(6)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	44.56533	16.89830	2.637267	0.0084
DCL(-1)	-0.044811	0.023032	-1.945610	0.0517
		Variance Equation		
C	2059.162	205.6106	10.01486	0.0000
RESID(-1)^2	0.059478	0.004606	12.91390	0.0000
RESID(-1)^2*(RESID(-1)<0)	-0.054683	0.004790	-11.41551	0.0000
GARCH(-1)	0.968345	0.002655	364.6968	0.0000

R-squared	0.003497	Mean dependent var	33.93834
Adjusted R-squared	0.002889	S.D. dependent var	1228.217
S.E. of regression	1226.441	Akaike info criterion	16.40016
Sum squared resid	2.47E+09	Schwarz criterion	16.41991
Log likelihood	-13450.33	Hannan-Quinn criter.	16.40749
Durbin-Watson stat	2.031985		

Source: Prepared by the researchers using Eviews 13

04 - Estimation of the EGARCH Model

Dependent Variable: DCL
Method: ML ARCH - Normal distribution (Marquardt / EViews legacy)
Date: 08/21/24 Time: 00:55
Sample (adjusted): 1/03/2020 6/30/2024
Included observations: 1641 after adjustments
Convergence achieved after 87 iterations
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1))) +
C(5)*RESID(-1)/@SQRT(GARCH(-1)) + C(6)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	52.44411	15.69927	3.340545	0.0008
DCL(-1)	-0.064143	0.023365	-2.745298	0.0060
		Variance Equation		
C(3)	0.037769	0.010129	3.728650	0.0002
C(4)	0.125358	0.009146	13.70557	0.0000
C(5)	0.035693	0.004991	7.152168	0.0000
C(6)	0.991104	0.000871	1138.227	0.0000

R-squared	0.003668	Mean dependent var	33.93834
Adjusted R-squared	0.003060	S.D. dependent var	1228.217
S.E. of regression	1226.336	Akaike info criterion	16.38872
Sum squared resid	2.46E+09	Schwarz criterion	16.40847
Log likelihood	-13440.94	Hannan-Quinn criter.	16.39604
Durbin-Watson stat	1.992631		

Source: Prepared by the researchers using Eviews 13

We must select the best model, for which we will analyze its parameters and rely on it for forecasting, based on certain criteria (AIC, SIC, H-Q). This is referred to as the Maximum Likelihood Estimation (M.L.E) method. We will clarify this in the following table:

Table 03: Selection of the Best Model

H-Q	**AIC**	**Sch**	**Models**
19.580	16.4113	16.4278	GARCH
20.346	16.4001	16.4199	TGARCH
19.717	16.3887	16.4084	EGARCH

Source: Prepared by the researchers using Eviews 13

From the estimation results, it is evident that the EGARCH model is the best model, as it has the lowest value among the aforementioned criteria.
After selecting the EGARCH model, we summarize its parameter estimates in the following table:

Table 04: Parameters of the EGARCH Model

Variable	**coefficient**	**Std.Error**	**z-Statistic**	**Prob**
C(3)	0.037769	0.01012	3.7286	0.0002
C(4)	0.125358	0.009146	13.7055	0.0000
C(5)	0.03569	0.004991	7.1521	0.0000
C(6)	0.991104	0.000871	1138.227	0.0000

Source: Prepared by the researchers using Eviews 13

From the table estimating the EGARCH model, we can derive the following equation:

LOG(GARCH) = 0.0377692956693 + 0.125357897273*ABS(RESID(-1)/@SQRT(GARCH(-1))) + 0.0356931393625*RESID(-1)/@SQRT(GARCH(-1)) + 0.991103680826*LOG(GARCH(-1))

We observe that all model parameters are statistically significant at the 1% level, indicating that the model is statistically acceptable.

- **C(4):** This parameter indicates the extent to which the size of the shock affects the variance of future fluctuations in Bitcoin.
- **C(5):** This parameter provides insight into how the sign of the shock impacts future fluctuations in Bitcoin.
- **C(6):** This parameter offers insight into the persistence of past fluctuations and how they help in predicting future fluctuations.
- **c(4)** is positive, indicating a positive relationship between past variance and current variance in absolute terms. This means that as the size of the shock increases, volatility also increases.
- **c(5)** is positive, indicating that good news will increase volatility more than bad news of the same magnitude—evidence of Bitcoin's sensitivity.

Model Diagnostic Tests: After diagnosing the appropriate model, it is essential to ensure its validity and efficiency by using the ARCH test on the residuals to verify that the model is free from ARCH effects. Additionally, the model will be examined using the Q-statistic to ensure that the residuals do not exhibit autocorrelation. We will illustrate this in the following tables:

- **Test for ARCH Effects on Model Residuals:**

Table 05: Results of the Breusch-Godfrey Serial Correlation LM Test.

Breusch-Godfrey serial correlation LM test			
F-statistic	1.7907	Prob.F(1,1638)	0.1810
Obs*R-squared	1.7910	Prob.Chi-square(1)	0.1808

Source: Prepared by the researchers using Eviews 13.

It is observed that the calculated F value reached 1.7907 with a probability of (0.1810), which is greater than 5%. Therefore, we accept the null hypothesis, indicating that there is no ARCH effect on the residuals.

Table 06: Results of the Q-statistic Test for the Residuals of the EGARCH Model.

Date: 08/21/24 Time: 00:56
Sample (adjusted): 1/03/2020 6/30/2024
Q-statistic probabilities adjusted for 1 dynamic regressor

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.060	0.060	5.9775	0.014
		2	0.034	0.031	7.8909	0.019
		3	0.037	0.033	10.166	0.017
		4	0.033	0.028	11.909	0.018
		5	0.018	0.012	12.416	0.030
		6	0.005	0.001	12.464	0.052
		7	0.001	-0.002	12.467	0.086
		8	-0.030	-0.032	13.951	0.083
		9	0.053	0.056	18.595	0.029
		10	-0.008	-0.013	18.703	0.044
		11	-0.005	-0.005	18.748	0.066
		12	0.013	0.013	19.028	0.088
		13	0.031	0.029	20.663	0.080
		14	0.017	0.013	21.159	0.098
		15	0.036	0.033	23.327	0.077
		16	-0.006	-0.015	23.386	0.104
		17	0.011	0.011	23.586	0.131
		18	0.015	0.007	23.981	0.156
		19	0.003	-0.000	23.992	0.196
		20	-0.023	-0.024	24.888	0.206
		21	-0.008	-0.006	24.983	0.248
		22	0.000	-0.001	24.983	0.298
		23	-0.024	-0.022	25.974	0.302
		24	0.051	0.052	30.345	0.174
		25	0.046	0.045	33.808	0.112
		26	0.027	0.020	35.012	0.111
		27	0.025	0.016	36.080	0.114
		28	0.040	0.029	38.804	0.084
		29	-0.038	-0.048	41.262	0.065
		30	0.012	0.009	41.490	0.079
		31	0.031	0.026	43.054	0.073
		32	-0.009	-0.008	43.198	0.089
		33	0.037	0.036	45.541	0.072
		34	0.006	0.000	45.604	0.088

Source**:** Prepared by the researchers using Eviews 13.

It is clear from the table above that there is no autocorrelation between the residuals, nor between the squares of the residuals across all periods. Therefore, the residuals are random and independently distributed for each model.

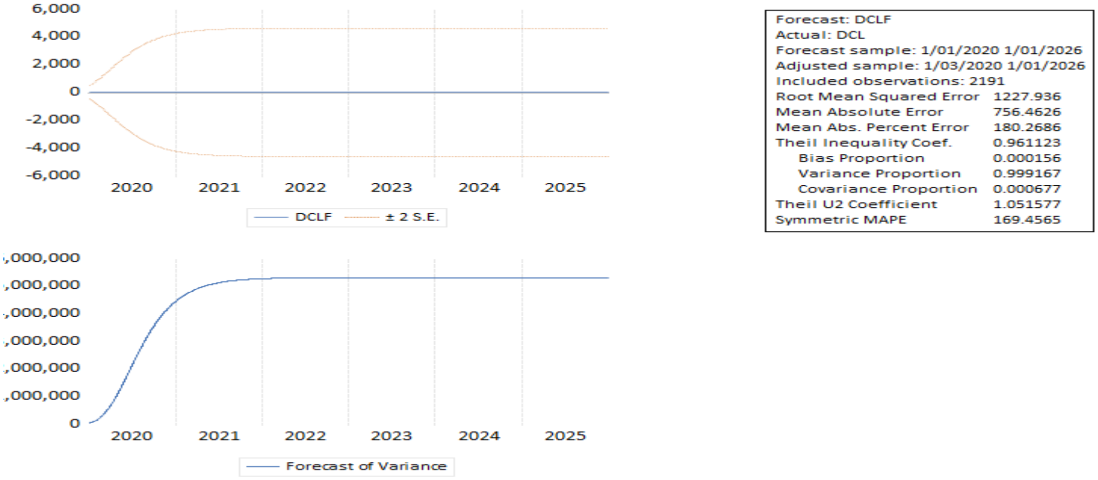

Figure 04: Forecast of the model values and the residuals of the model.
Source: Prepared by the researchers using Eviews 13.

As a final step, we predicted the values of the optimal model, as shown in the previous table. It became evident that the estimates were within the confidence limits at a level of 0.95%. The quality of the estimates for the model is also reflected in the root mean squared error (RMSE), which was the lowest among the models tested, with a value of (RMSE = 1227.936). Additionally, the mean absolute error (MAE) was (MAE = 756.4626), and the Theil inequality coefficient was (Theil = 0.9611), which is less than one.

Conclusion

To apply the conditional autoregressive models with heteroskedastic errors to the daily price series of Bitcoin from January 1, 2020, to June 30, 2024, it was essential to ensure the stability of the series under study. This was achieved by removing the trend component and ultimately obtaining the DCL series. Various statistical tests (ADF; PP) confirmed the stability of the series.
From the stages of modeling the DCL series, we can conclude the following results:

- Bitcoin prices experienced significant fluctuations during the study period.
- Diagnostic tests confirmed that the DCL series can be represented by ARCH errors.
- The best standard model for estimating the time series data of Bitcoin prices is the EGARCH model among other ARCH models, as this model has the lowest values for the statistical criteria used (H-Q, SIC, AIC) when comparing the studied models.
- One of the main drawbacks of symmetric ARCH models is their focus on the symmetry property of phenomena. To address this gap, asymmetric ARCH models were introduced, which consider the characteristics of asymmetric phenomena.

Recommendations:

Based on this study, we recommend that all interested parties increase their use of heterogeneous models due to their ability to capture volatility (risks). We also advise careful selection of the appropriate model based on the shape and characteristics of the time series data to avoid errors in estimation and prediction processes. It is preferable to use relatively long periods for the data of the series under study to reveal the serial correlation and non-stationarity of the conditional variance among the observations of the series in question.

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